

Introduction

The UK Met Office's operational global forecasts are currently based on a hybrid four-dimensional variational assimilation (hybrid-4DVar). In this scheme, the background error covariance (\mathbf{B}) at the beginning of the assimilation window is specified as a weighted sum of a fixed (but full-rank) climatological part and a flow-dependent (but limited-rank) contribution derived from the MOGREPS-G ensemble. However, the perturbation-forecast (PF) and adjoint models which 4DVar uses to evolve covariances over time impose significant computational and maintenance cost, and may not scale well on future massively-parallel computer systems. The Met Office is therefore testing an alternative scheme, called 4D-Ensemble-Var (4DEnVar), in which the temporal correlations are taken from the ensemble, whilst the climatological part of the hybrid reverts to a three-dimensional formulation.

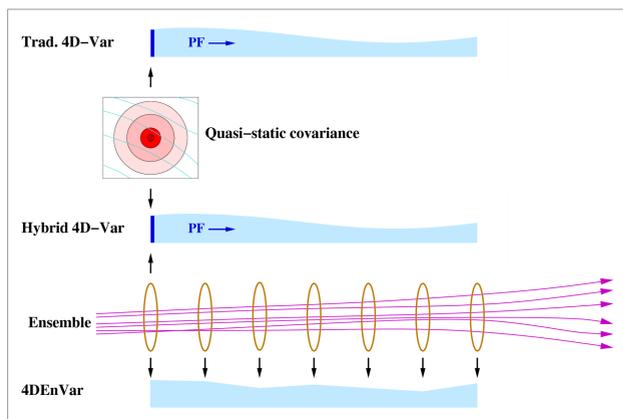


Figure 1: Traditional 4DVar (top) defines a fixed \mathbf{B} at the beginning of the assimilation window, which is evolved and made partially flow-dependent using PF and adjoint models. Hybrid 4DVar (middle) augments the climatological \mathbf{B} with a flow-dependent contribution from a separate ensemble, but still uses PF and adjoint models to evolve this in time. 4DEnVar (bottom) uses the 4D covariance predicted by the ensemble, together with a 3D climatological contribution, so no PF or adjoint models are required. In the current implementation, a single set of ensemble member coefficients is chosen for the whole assimilation window, so there is no time localisation (only spatial localisation).

Impact on deterministic forecasts

Several trials have been run to compare the performance of deterministic forecasts initialised using 4DEnVar with other approaches. Notable features of the trial setup include:

- Analysis increments calculated at N216 resolution (60km typical grid spacing).
- Ensemble covariances from the operational 44-member MOGREPS-G ensemble, again at N216 resolution, which calculates initial perturbations using a local Ensemble Transform Kalman Filter (ETKF), centred around the analysis from the existing operational hybrid-4DVar.
- Covariance weights: 0.8 climatological, 0.5 ensemble (sum exceeding 1.0 to make the fit to observations comparable to non-hybrid 4DVar; our operational system now uses 1.0, 0.3).
- N320 Deterministic forecasts (40km typical grid spacing).
- Northern Hemisphere autumn period (9 Oct–8 Nov 2011).
- Gaussian horizontal localisation with half-width 1200km.
- Vertical localisation as in the operational hybrid-4DVar.
- Initialisation uses 6h Incremental Analysis Update (IAU) for 3DVar. An 'IAU-like' approach filters the climatological but not the ensemble modes in 4DEnVar, whilst 4DVar uses a J_c balance penalty term instead of an IAU.

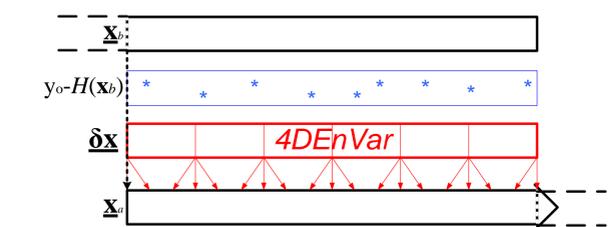


Figure 2: 3DVar uses a standard IAU in which the single increment is split over a 6h window to filter out high-frequency features which are presumed to be noise. 4DEnVar deduces an increment which combines a constant 3DVar-like climatological contribution with an evolving ensemble contribution. The 'IAU-like' application of this increment again filters high-frequency features from the climatological contribution, together with time-uncorrelated noise from the ensemble data, but leaves the coherent ensemble evolution unfiltered.

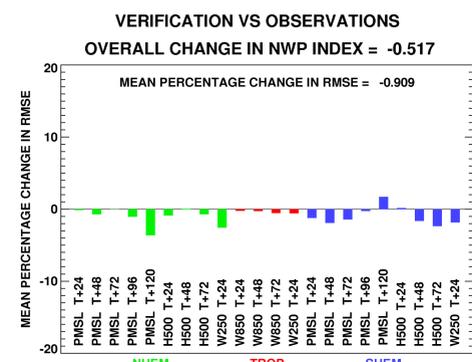


Figure 3: Performance of 4DEnVar compared to hybrid-3DVar. These have similar background error specifications, except that hybrid-3DVar ignores the time dimension of the ensemble data. The general reduction in RMS error indicates that this time evolution is generally beneficial.

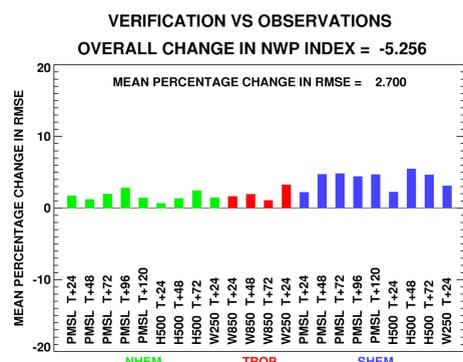


Figure 4: Performance of 4DEnVar compared to hybrid-4DVar (representing the performance of the current operational system degraded to the same analysis and forecast resolution as the other trials). Possible reasons for the inferior performance of 4DEnVar include the loss of flow-dependence in the climatological part of the hybrid, the fact that the ensemble localisation does not move with the flow, and initialisation issues. These are explored more fully in Andrew Lorenc's talk.

Control	RMSE	NH	Tropics	SH
hybrid-3DVar ⁴⁴	-0.9%	10/113/0	0/123/0	7/112/4
hybrid-4DVar ⁴⁴	+2.7%	0/100/23	0/102/21	0/44/79

Table 1: Summary of the performance of 4DEnVar for a larger set of variables, levels and lead-times (each combination is termed a 'component'). The right-hand columns show the number of components for which the 4DEnVar RMS error is better/neutral/worse than the specified control, where 'neutral' is defined as $\pm 2\%$. The 4DEnVar advantage over hybrid-3DVar is particularly apparent in the Northern Hemisphere, whilst the disadvantage compared to hybrid-4DVar is stronger in the Southern Hemisphere.

Future work

We are currently investigating the impact of waveband localisation on 4DEnVar. Flow-adaptive localisation may also help to improve its performance. Trials with increased weight on the ensemble covariances, even if not optimal, may simplify the interpretation of results by making the methods more comparable.

The reduced cost of 4DEnVar makes it an attractive method for ensemble initialization. It has theoretical advantages over the ETKF currently used by MOGREPS-G, in areas such as localisation, re-linearization, the use of balanced variables, and greater consistency with the way the central analysis is produced (Bowler *et al.*, 2013). A single system serving both purposes should also reduce maintenance costs. Trials of a 4DEnVar-based ensemble are currently ongoing. We may also test a system using perturbed observations instead of the deterministic filter approach, since this would remove the assumption that the DA scheme is optimal. Better ensemble perturbations should in turn benefit the hybrid DA, further improving both the deterministic and ensemble forecasts.

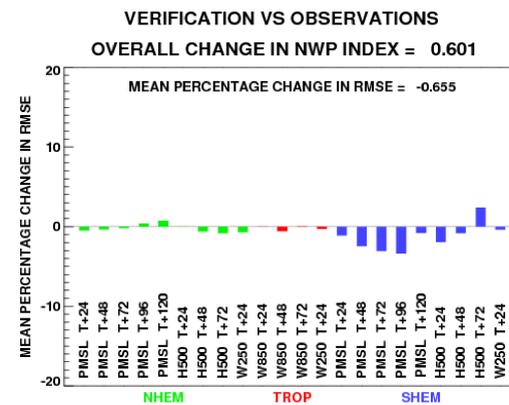


Figure 5: The use of a larger 176-member ensemble reduces the RMS error from 4DEnVar. Both sides of this comparison use covariance weights of 1.0 climatological, 0.3 ensemble, and localisation as above.

Idealised localisation experiments

Another focus of future work will be the interaction of ensembles and DA at convection-permitting scales. Appropriate localisation may be key to the success of convective-scale ensemble DA, due to the complex, variable dynamics and wide range of scales. Dong *et al.* (2011) present a situation in which the assimilation is improved by using different localisation radii for different observation types. I have conducted some idealised localisation experiments to explore this surprising result, whilst avoiding confounding issues such as non-Gaussian statistics, different observations affecting different variables, and the cyclic dependence of \mathbf{B} on past observations.

The basic scenario considers 100 points on a 1D line. A uniform \mathbf{B} is specified analytically, with unit variance and a Gaussian falloff as a function of distance with a half-width of 5 gridpoints. Both the truth and 5 background ensemble members are obtained as random samples from this covariance using an eigenvector decomposition. Observations are provided at specified intervals with unit error variance. The scenario is repeated 1000 times to estimate the RMS error of the ensemble mean analysis, which is obtained by direct application of the Kalman Filter equation.

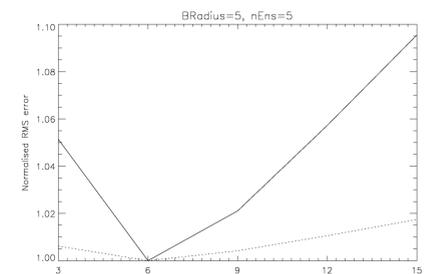


Figure 6: RMS error as a function of the half-width of the Gaussian localisation function, for scenarios with observations every 2 (solid) or 40 (dotted) gridpoints. Each line has been normalised by its minimum value to highlight the impact of localisation radius despite the large difference in RMS error between the two different observation densities. In this simple case, both observation spacings are optimised by very similar localisation radii.

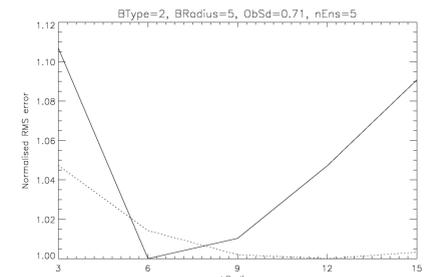


Figure 7: As Figure 6, except that \mathbf{B} is now an equal-weight sum of Gaussians with half-widths of 5 and 20 gridpoints, representing a multi-scale system. The localisation is still a simple Gaussian. The optimal localisation radius for the sparse observations is now about twice that for the dense network, even though \mathbf{B} is the same in both cases. The observation error variance has been halved to emphasise the result.

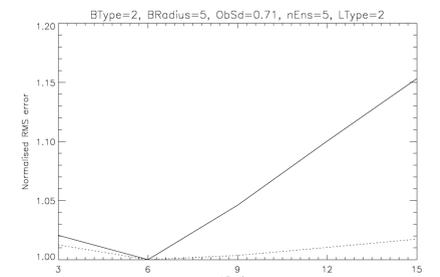


Figure 8: As Figure 7, except the localisation is now an equal-weight sum of Gaussians with half-widths L Radius and $4 \cdot L$ Radius. The optimal radius is now very similar for the two observation densities (although the RMS error for dense observations is marginally worse than the best simple Gaussian – not shown). This suggests that the major problem in Figure 7 may be not so much the multi-scale \mathbf{B} as the failure of the localisation to respect this new shape. This suggests a potential advantage for adaptive schemes that derive the localisation as a statistically-motivated function of the sample \mathbf{B} .

References

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- Dong J, Xue M, Drogemeier K. 2011. The analysis and impact of simulated high-resolution surface observations in addition to radar data for convective storms with an ensemble Kalman filter. *Meteorol. Atmos. Phys.* **112**: 41–61.