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A Hybrid Variational-Ensemble Data Assimilation Method with an Implicit Optimal Hessian Preconditioning

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Outline

- Hybrid variational-ensemble methods
- Hessian preconditioning and static error covariance
- Hybrid data assimilation with WRF model and real observations
- Future development

HYBRID VARIATIONAL-ENSEMBLE DATA ASSIMILATION

Take advantage of both variational and ensemble DA methodologies

Hybrid methods generally address two major aspects:

(1) Error covariance

- flow-dependence
- rank
- uncertainty feedback

Combine flow-dependent and static error covariance

(2) Nonlinearity

- iterative minimization
- Hessian preconditioning

Use iterative minimization to obtain optimal analysis solution

Practical issues of hybrid data assimilation

Combine flow-dependent and static error covariance

1- Linear combination of full matrices or square-root matrices

$$P_f^{1/2} = \alpha P_{ENS}^{1/2} + \left(1 - \alpha\right) P_{VAR}^{1/2}$$

2- What is the optimal way of combining static and flow dependent matrices?

Use iterative minimization to obtain optimal analysis solution

1- Iterative minimization from variational methods

2- Can this be improved by using an independent iterative minimization with optimal Hessian preconditioning?

[*Note*: Optimal Hessian preconditioning is defined here as an inverse square-root of the Hessian matrix (e.g., Axelsson and Barker 1984)]

$$G = EE^T \Rightarrow G^{-1/2} = E^{-T}$$

Limitations of optimal Hessian preconditioning in hybrid data assimilation

- Assume standard cost function $$J(x)=\frac{1}{2}[x-x^f]^T P_f^{-1}[x-x^f]+\frac{1}{2}[y-h(x)]^T R^{-1}[y-h(x)]$$
- Apply common change of variable $$x^a = x^f + P_f^{1/2} w$$
- Optimal Hessian preconditioning is $$G^{-1/2}=\left(I+P_f^{T/2}H^T R^{-1} H P_f^{1/2}\right)^{-1/2}$$

In ***variational*** data assimilation the inversion is practically impossible due to high dimension of state ($N_s \sim 10^7$) and static error covariance matrix (N_s x N_s)

In ***ensemble*** data assimilation the inversion is possible due to reduced rank ensemble error covariance, implying the preconditioning matrix of smaller size (N_{ens}xN_{ens})

In ***hybrid*** data assimilation the inversion is limited by requirements of the (full-rank) static error covariance

- ***Option #1***: variational framework (use preconditioning from variational methods)
- ***Option #2***: ensemble framework (**define reduced-rank static error covariance** first, then use preconditioning from ensemble methods)

If feasible, the option #2 allows optimal Hessian preconditioning in hybrid data assimilation methods

Reduced-rank static error covariance

1. Assume that a full rank static error covariance square root has been defined

$$P^{1/2}$$

2. Construct an orthonormal reduced rank matrix *Q*, and

3. Define a reduced-rank static covariance P_{RR} as

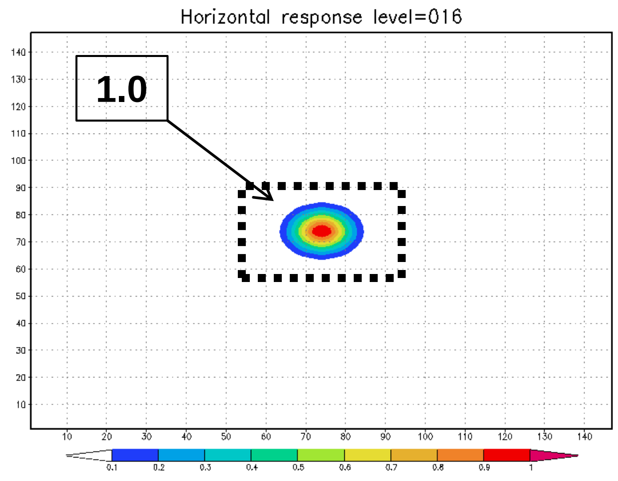
$$P_{RR}^{1/2} = P^{1/2}Q$$

How to define *Q*?

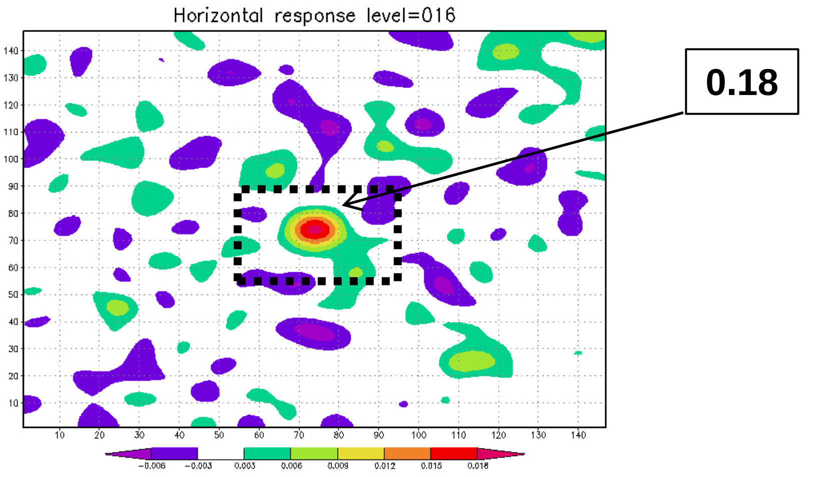
1. Use SVD of local matrix and truncate (preserve similarity with global matrix)
2. Build *global* block-circulant matrix from *local* singular vectors (preserve orthogonality)
3. Scale by diagonal matrix to account for SVD truncation

Processing reduced rank matrix:
GLOBAL HORIZONTAL RESPONSE TO A SINGLE OBSERVATION

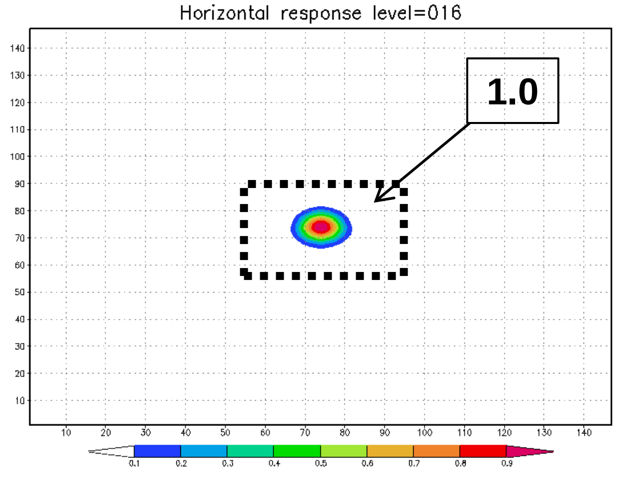
Horizontal response (truth)



Horizontal response (RR)



Horizontal response (RR + localization + scaling)



Sufficient rank covariance becomes acceptable after post-processing

Preliminary assessment the proposed hybrid methodology: Experimental design

- *Model*: **WRF-ARW mesoscale model at 27 km / 28 layer resolution**

  - 80 x 75 x 28 grid points

- Control variables: wind, perturbation potential temperature, specific humidity
- *DA algorithm*: **Maximum Likelihood Ensemble Filter (MLEF)**

  (1) ***static***: Reduced rank static forecast error covariance with **40** columns/ensembles

  (2) ***dynamic***: Standard ensemble algorithm with **32** ensembles

  (3) ***hybrid***: Combined static and dynamic forecast error covariance with **72** columns/ ensembles

- *Observation operator*: **Forward component of Gridpoint Statistical Interpolation (GSI)**

  - NCEP operational observations and quality control

- *Experimental setup*:

  - May 20, 2013, central United States

  - 6-hour assimilation window

  - Linear combination coefficient $a = 0.7$

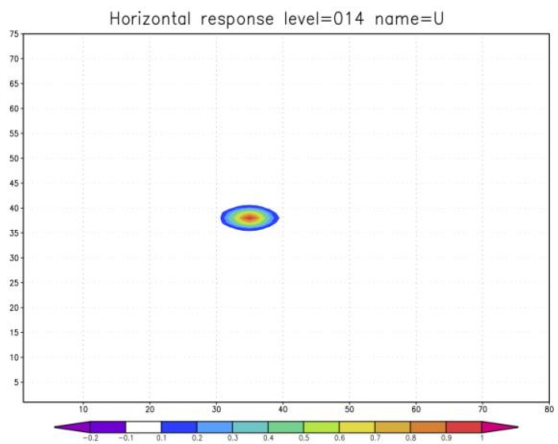
Full rank static error covariance

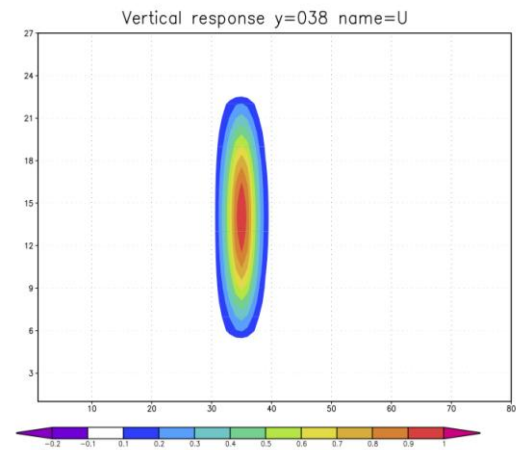
- *Toeplitz matrix as a covariance for stationary process*
- *Simplified cross-correlations between variables*

Horizontal | ***Vertical***

Variable 1:

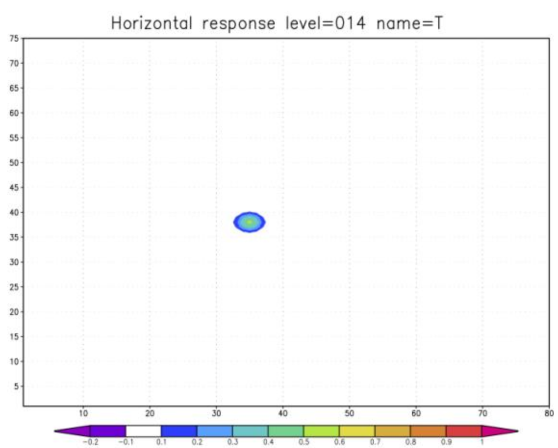
Auto-correlation

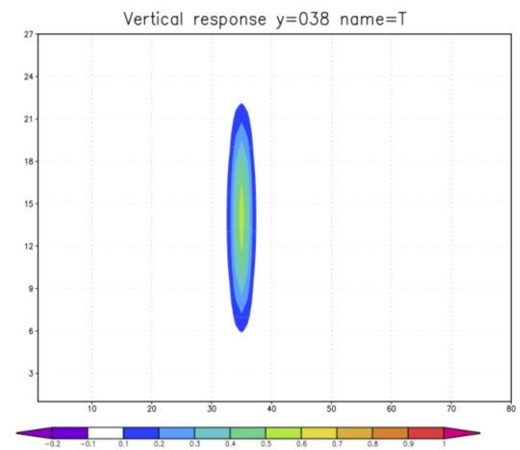




Variables 2,3,4:

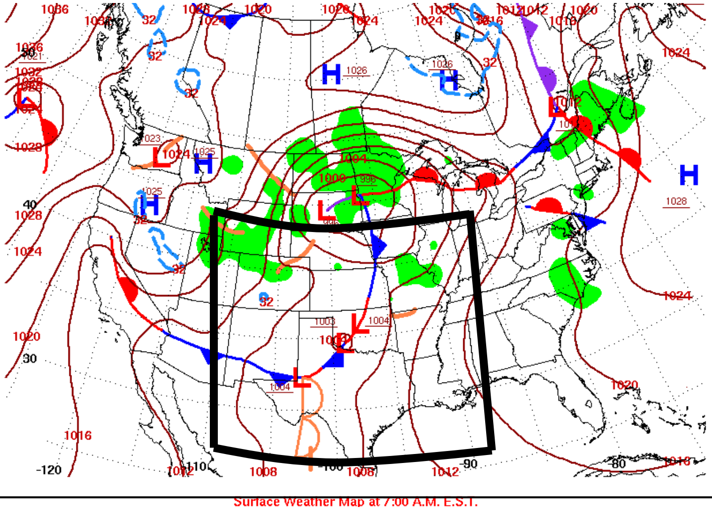
Cross-correlation





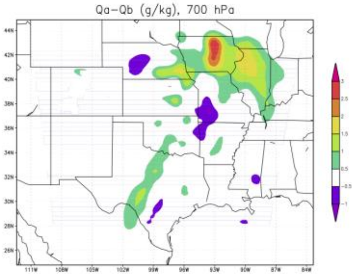
Synoptic situation

- ***Severe weather with tornadoes over Oklahoma***
- ***Front associated with a low in upper midwest***

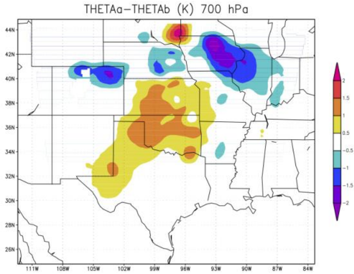


Surface weather map valid 1200 UTC on May 20, 2013

Specific humidity (700 hPa)



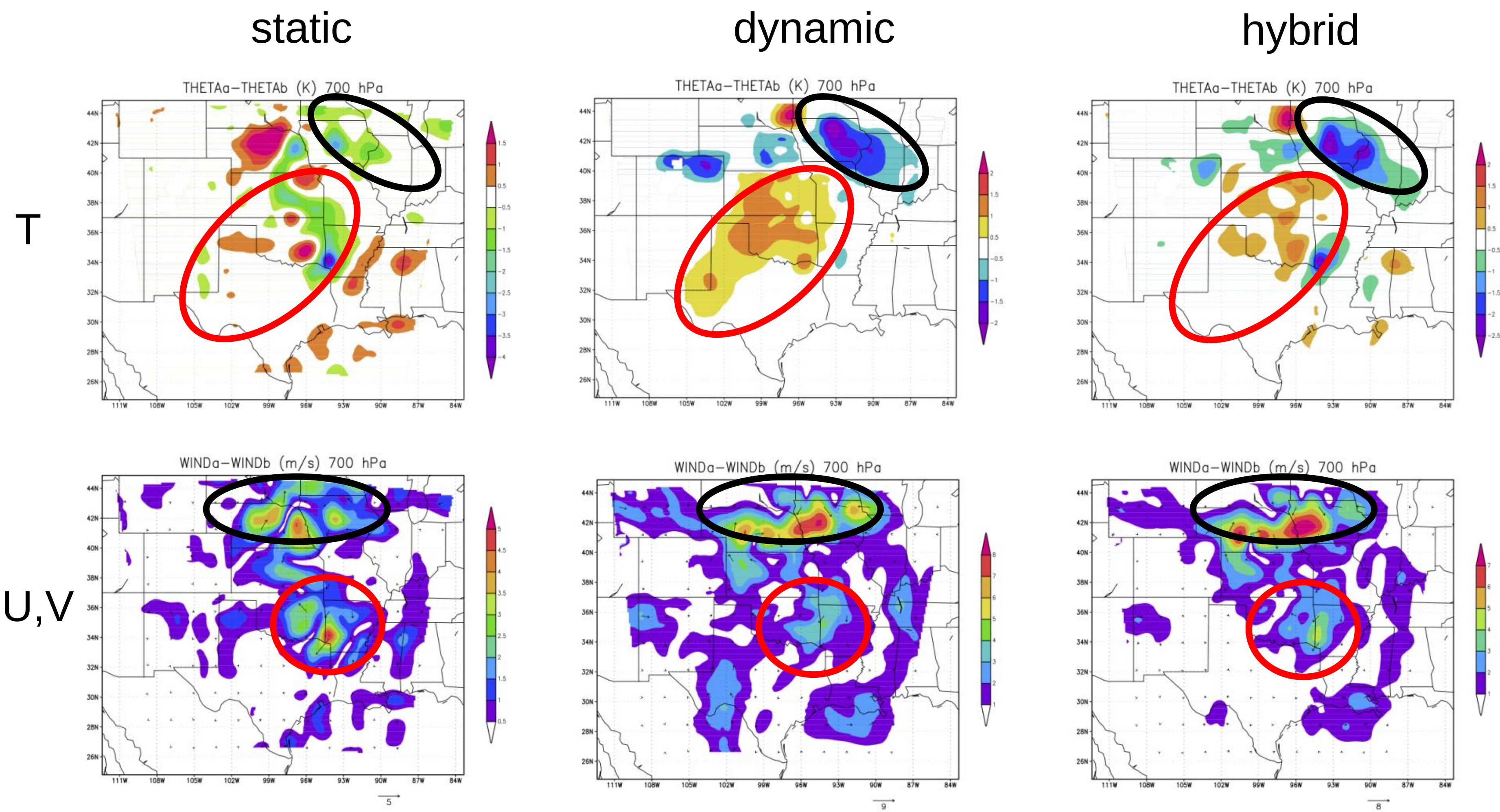
Temperature (700 hPa)



Analysis increments (x^a-x^f) of standard MLEF (32 ensembles) show dominant analysis adjustments along the front

Experiment 2: Analysis increments (x^a-x^b) at 700 hPa

(valid 00 UTC 20 May 2013)



Hybrid produces a mixture of dynamic and static information: either one can prevail locally

Summary and future work

- **Proof of concept that the presented hybrid system can work with cross-covariances**
- **Reduced rank static error covariance approach may be feasible for realistic applications – allows optimal Hessian preconditioning**
- **Preliminary experiments with new hybrid system encouraging**
  - realistic model
  - real data
- **The anticipated performance has been achieved**

- **Future improvements of reduced rank static error covariance**
  - high-dimensional state and realistic variational covariance
  - examine alternative bases: Fourier, wavelet
- **Future improvements of mixing static and dynamic information**
  - diagonal matrix instead of alpha (e.g., augmented control variable)
  - define orthogonally complement subspaces
- **Tests new hybrid method in realistic weather systems**
  - all-sky satellite radiance assimilation
  - coupled land-atmosphere-chemistry models