

Posterior covariance and analysis error covariance in variational data assimilation

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The problem of variational data assimilation for a nonlinear evolution model is stated as an optimal control problem to estimate the analysis (initial condition) [1]. The data contain errors (observation and background errors), hence there is an error in the analysis. For mildly nonlinear dynamics, the analysis error covariance can be approximated by the inverse Hessian of the cost functional in the auxiliary data assimilation problem [2], whereas for stronger nonlinearity the 'effective' inverse Hessian approach may be used [3, 4]. However, the analysis error covariance is not the posterior covariance from the Bayesian perspective. While these two are equivalent in the linear case, the difference may become significant for nonlinear problems [5]. For the proper Bayesian posterior covariance a new approximation via the Hessian of the original cost functional is derived and its 'effective' counterpart is introduced. An approach for computing the analysis error covariance and posterior covariance in the matrix-free environment using Lanczos method with preconditioning is suggested. Numerical examples which validate the developed theory are presented for the model governed by the Burgers equation with a nonlinear viscous term.

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