4D-En-Var : link with weak-constraint 4D-Var and different possible implementations

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The 4D-Var formulation is currently used by most operational Numerical Weather Prediction centres. However, background error covariances have to be modelled at the initial time of the 4D-Var assimilation period, and their space and time variations are often approximated. Furthermore, the 4D-Var is known to be poorly scalable on computers with a large number of processors, due to its use of low-resolution versions of the linearized model.

The Ensemble Kalman Filter (EnKF) formalism has received a great attention and has initiated a lot of work on the use of ensembles in data assimilation. While the use of localization in observation space raises issues (e.g. for the use of satellite data), it allows indeed background error covariances to evolve in space and time. It was also shown that the EnKF can be combined with the variational approach.

The 4D-En-Var formulation has also received a considerable attention in recent years. The interest in this formulation relies on different nice properties: it allows 4D flow-dependent background error covariances to be represented, it avoids (in contrast to EnKF) the localization of background error covariances in observation space, it is potentially highly parallel on new computer architectures, and it also gets rid of the development, maintenance and cost of tangent-linear and adjoint models.

The aim of this paper ([1], [2]) is to point out the link between the weak-constraint 4D-Var formulation and its 4D-En-Var counterpart. Furthermore, the 4D-En-Var formulation is relatively easy to precondition, and it can potentially include a larger class of representation of model errors. The paper also aims at showing that the so far proposed implementations of the 4D-En-Var implicitly rely on a preconditioning of the variational problem using the square-root of the localized 4D ensemble covariance matrix $\mathbf{B}^e$. Two other possible formulations are proposed, both relying on a preconditioner given by the complete $\mathbf{B}^e$ matrix. One of them performs the minimization in the dual space (with a size given by the number of observations).

It is shown that the avoidance of the need to manipulate the square-root of the $\mathbf{B}^e$ matrix allows a larger flexibility in the specification of the covariance localization needed in 4D-En-Var. The use of a hybrid covariance matrix combining a modelled $\mathbf{B}^c$ matrix and an ensemble $\mathbf{B}^e$ matrix is also discussed. An application of the proposed implementations of 4D-En-Var is shown with the Burgers' model and compared to the use of 4D-Var.

References

[1] G. Desroziers, J.-T. Camino and L. Berre. "A possible implementation of the 4D-Var based on a 4D-ensemble", ***Presentation at the International Conference on Ensemble Methods in Geophysical Sciences*****, Toulouse, France, 12-16 November. Available online at** **www.meteo.fr/cic/meetings/2012/ensemble.conference/presentations/session06/2.pdf .**

[2] G. Desroziers, J.-T. Camino and L. Berre. "4D-En-Var: link with weak-constraint 4D-Var and different possible implementations", ***Q. J. R. Meteor. Soc.*****, submitted.**